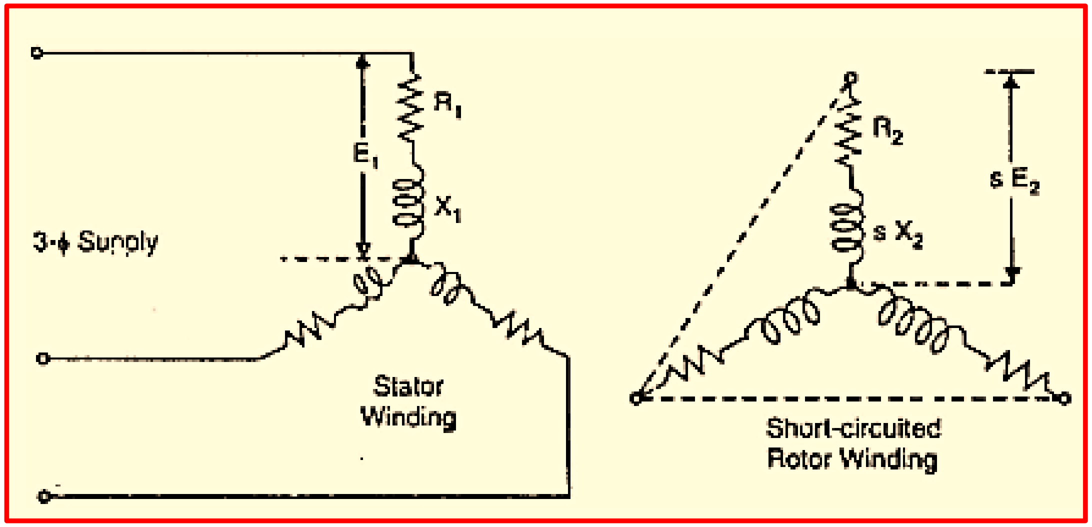
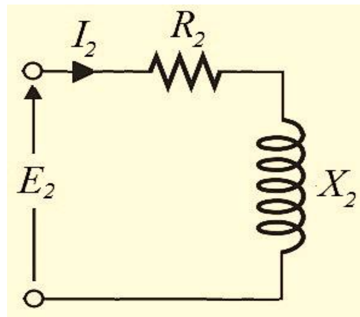
Rotor Current



- The figure shows the circuit of a 3-phase induction motor at any slip *s*. The rotor is assumed to be of wound type and Y- connected. *Note that*: the rotor e.m.f/phase and the rotor reactance/phase are sE_2 and sX_2 respectively. The rotor resistance/phase is R_2 (not depend on the frequency) therefore, it does not depend upon the slip. Likewise, stator winding values R_1, E_1, and X_1 do not depend upon the slip.
- *Since the motor represents a balanced 3-phase load, we need consider one phase only; the conditions in the other two phases being similar.*

Rotor Current at standstill

Figure shows one phase of the rotor circuit at standstill.



[here: *s* = 1]

Rotor current/phase,

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

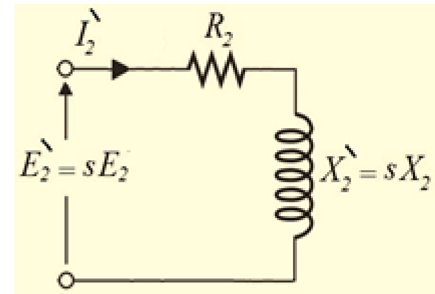
Rotor power factor (*pf*), $\cos\phi_2$

$$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

ϕ_2 : angle between R_2 and Z_2

Rotor Current When Motor Runs at slip *s*

Figure shows one phase of the rotor circuit when the motor is running at slip *s*.



[here: *s* <1]

Rotor current/phase,

$$I_2^{\backslash} = \frac{E_2^{\backslash}}{Z_2^{\backslash}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Rotor power factor (*pf*), $\cos\phi_2^{\backslash}$

$$\cos\phi_2^{\backslash} = \frac{R_2}{Z_2^{\backslash}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$\phi_2^{\backslash}$: angle between $R_2^{\backslash}$ and $Z_2^{\backslash}$

Rotor Torque of Induction Motor (T)

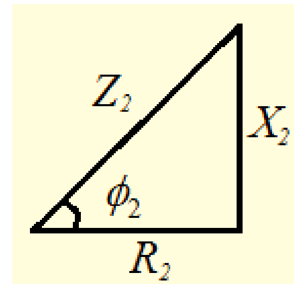
- It has be shown that in the case of a dc motor, the rotor torque is proportional to the product of armature current and the flux per pole. Similarly in the induction motor, the torque is also proportional to the product of the rotor current and the flux per stator pole. However, there is one more factor that has to be taken into account (*i.e.*, the power factor of the rotor).
- In dc motor : $T \ \alpha \ \Phi I_a$
- In induction motor : $T \ \alpha \ \Phi I_2 \cos\phi_2$ $\rightarrow$ eqn. 1
- In the induction motor : $E_2 \ \alpha \ \Phi$ $\rightarrow$ eqn. 2

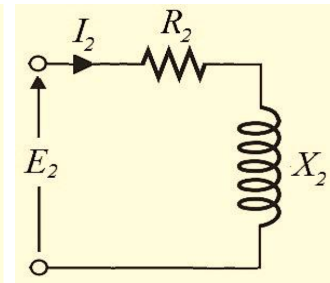
From the two equations : $\therefore T \ \alpha \ E_2 I_2 \cos\phi_2$

Here the torque (T) is the motor torque/phase

Rotor Torque of Induction Motor (T) …

- The torque/phase developed by the rotor is proportional to:
  - i- Rotor current/phase.
  - ii- Rotor e.m.f/phase.
  - iii- Power factor of the rotor circuit.





$$\therefore T \;\alpha\; E_2 I_2 \cos\phi_2$$

$$T = K E_2 I_2 \cos\phi_2$$

Where;

E_2 : rotor e.m.f/phase at standstill (*volts*)

I_2 : rotor current/phase at standstill (*ampere*)

ϕ_2 : angle between R_2 and Z_2

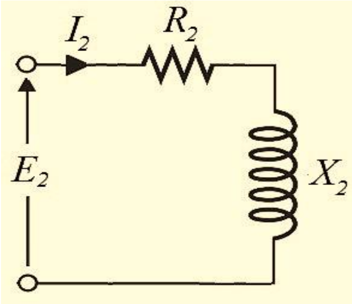
$\cos\phi_2$: rotor power factor at standstill

K : constant

T : the rotor (or motor) torque/phase (*Newton.meter*)

Starting Torque in Induction Motor (T_s)

- It is the torque developed by the motor at starting (or standstill). From figure, the rotor circuit/phase at standstill



- Let,

  E_2 : rotor e.m.f/phase at standstill (*volts*)

  X_2 : rotor reactance/phase at standstill (*Ohm*)

  R_2 : rotor resistance/phase at standstill (*Ohm*)

  $Z_2 = \sqrt{R_2^2 + X_2^2}$: rotor impedance/phase at standstill (*Ohm*)

Rotor current/phase, $$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$ ……… at standstill

Rotor *p.f*, $$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$ ……………. at standstill

Starting Torque in Induction Motor (T_s) …

- Then the starting torque/phase (or the standstill toque/phase) is:

$$T_s = K E_2 I_2 \cos\phi_2$$

$$= K E_2 \cdot \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$= \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

$$= \frac{K E_2^2 R_2}{Z_2^2}$$

$\underline{N_s \text{ is synch. speed } (rps)}$

$$if\ \ K = \frac{1}{2\pi N_s} \quad \rightarrow \quad T_s = \frac{1}{2\pi N_s} \cdot \frac{E_2^2 R_2}{Z_2^2} \qquad \text{........ torque per phase}$$

$$if\ \ K = \frac{3}{2\pi N_s} \quad \rightarrow \quad T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{Z_2^2} \qquad \text{........ motor starting torque}$$

Starting Torque in Induction Motor (T_s) …

- If the supply voltage (V) is constant, then the flux per pole (Φ) set up by the stator and hence the rotor e.m.f/phase (E_2) are both constants.

$$T_s = K E_2 I_2 \cos\phi_2$$

$$= K \cdot const \cdot \frac{const}{\sqrt{R_2^2 + X_2^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$= \frac{K_1 R_2}{R_2^2 + X_2^2}$$

$$= \frac{K_1 R_2}{Z_2^2}$$

Where: the constant K not equal to K_1

Condition for Maximum Starting Torque

- To get the maximum value of the starting torque, take a derivative for it w.r.t the constant R_2 and equating the result with zero as follows:

$$\because T_s = \frac{K_1 R_2}{R_2^2 + X_2^2}$$

$$\therefore \frac{dT_s}{dR_2} = K_1\left[\frac{(1)\times(R_2^2 + X_2^2) - (2R_2)\times(R_2)}{(R_2^2 + X_2^2)^2}\right] = 0$$

$$\therefore R_2^2 + X_2^2 - 2R_2^2 = 0$$

$$\therefore R_2 = X_2$$

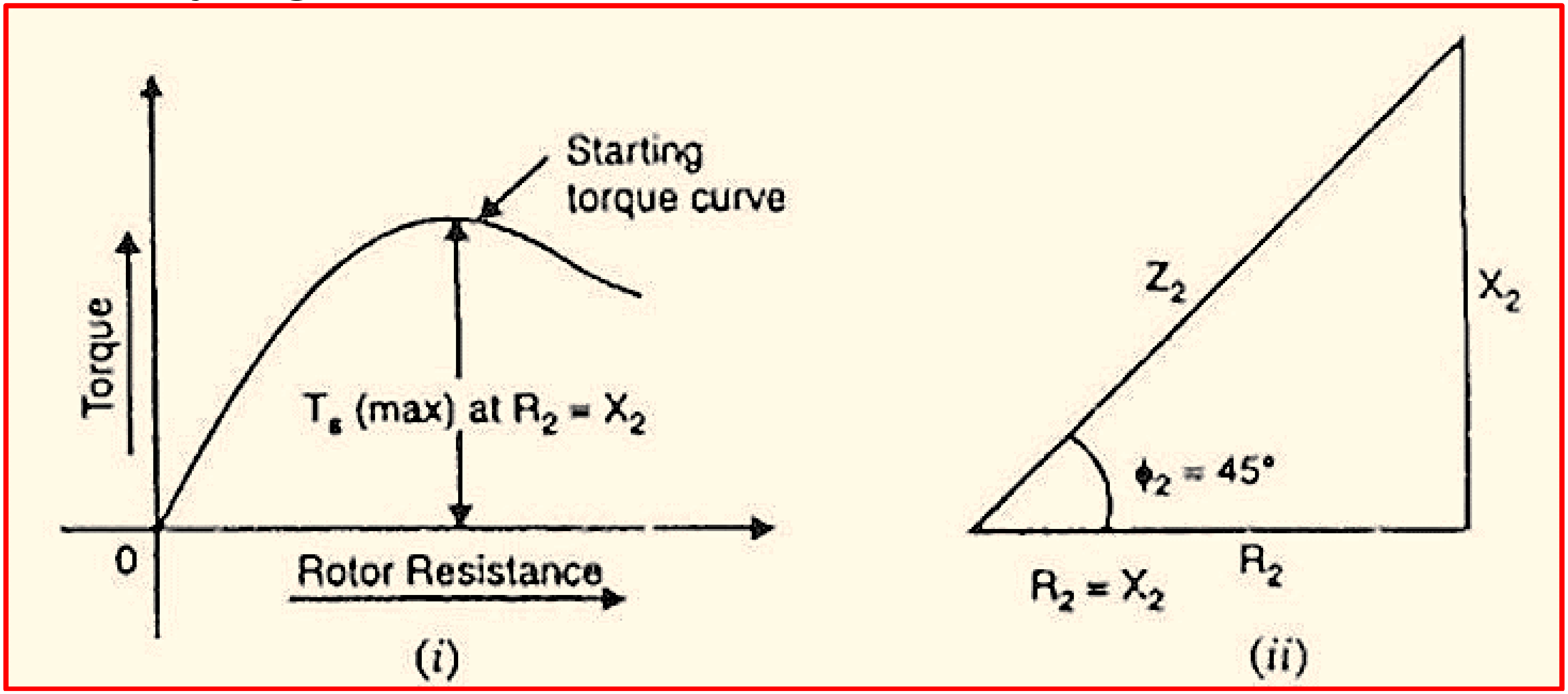
Hence the starting torque will be maximum when:

Rotor resistance/phase = Standstill rotor reactance/phase

$$T_{s(\max)} = \frac{K_1}{2R_2}$$

Condition for Maximum Starting Torque …

- Under the condition of maximum starting torque, $\phi_2 = 45°$ and rotor power factor is 0.707 lagging [See Figure].
- Lagging pf means the rotor current lags the induced e.m.f. of the rotor by angle $\phi_2 = 45°$.



Effect of Change of Supply Voltage on Starting Torque

$$\because T_s = \frac{KE_2^2 R_2}{R_2^2 + X_2^2}$$

Since, (E_2) α (supply voltage V) →

$$\therefore T_s = \frac{K_2 V^2 R_2}{R_2^2 + X_2^2}$$

Where K_2 is another constant

$$\therefore T_s \, \alpha V^2$$

Therefore, the starting torque is very sensitive to changes in the value of supply voltage. For example, a drop of 10% in supply voltage will decrease the starting torque by about 20 %. This could mean the motor failing to start if it cannot produce a torque greater than the load torque plus friction torque.

Why Starting Torque of 3-Phase Induction Motor is Low?

Really, the rotor circuit of an induction motor has low resistance and high inductance. At starting, the rotor frequency is equal to the stator frequency (*i.e.*, 50 Hz) so that the rotor reactance is large compared with rotor resistance. Therefore, rotor current lags the rotor e.m.f by a large angle, the power factor is low and consequently the starting torque is small.

$$T_s = K E_2 I_2 \cos\phi_2$$

At motor starting, the value of power factor ($cos\,\phi_2$) in Fig. 2 is less than its value in Fig. 1, and consequently the starting torque is small.

When resistance is added to the rotor circuit, the rotor power factor is improved which results in improved starting torque. This, of course, increases the rotor impedance and, therefore, decreases the value of rotor current but the effect of improved power factor predominates and the starting torque is increased.

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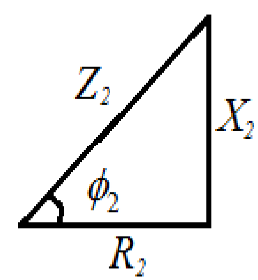


Fig. 1

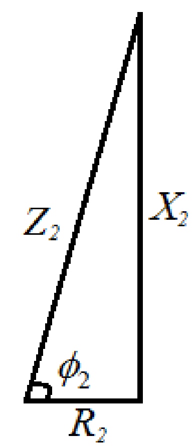


Fig. 2

Why Starting Torque of 3-Phase Induction Motor of Squirrel-cage Rotor Type Less than Wound Rotor Type?

(i) ***Squirrel-cage motors***: since the rotor bars are permanently short-circuited, it is not possible to add any external resistance in the rotor circuit at starting. Consequently, the starting torque of such motors is low.

(ii) ***Wound rotor motors***: the resistance of the rotor circuit of such motors can be increased through the addition of external resistance. By inserting the proper value of external resistance (to obtain $R_2 = X_2$), maximum starting torque can be obtained. As the motor accelerates, the external resistance is gradually cut out until the rotor circuit is short-circuited on itself for running conditions.

Example 3: A 3-phase induction motor having a star-connected rotor has an induced e.m.f of 80 volt between slip-rings at standstill on open-circuit. The rotor has resistance and reactance per phase of 1 Ω and 4 Ω respectively. Calculate, current/phase and power factor when:

(i) Slip-rings are short-circuited
(ii) Slip-rings are connected to star-connected rheostat of 3Ω per phase. Comment on the results

Answer

[Ans.: (i) 11.2A – 0.243 (ii) 8.16A – 0.707]

$$T_{s1} \alpha \, I_2 \cos\phi_2 ...... \frac{T_{s2}}{T_{s1}} = \frac{8.16 * 0.707}{11.2 * 0.243} = 2.12$$

Hence, the starting torque is increased due to the improvement in p.f. It will also be noted that improvement in p.f is much more than the decrease in current due to increased impedance.

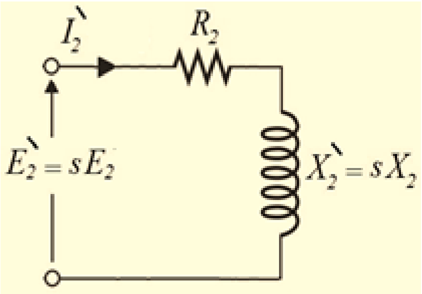
Example 4: A 3-phase, 400V, Y-connected induction motor has a Y-connected rotor. It has a stator to rotor turns ratio of 6.5. The rotor resistance and standstill reactance per phase are 0.05 Ω and 0.25 Ω respectively. What should be the value of external resistance per phase to be inserted in the rotor circuit to obtain maximum torque at starting? and What will be the rotor starting current and p.f at using this resistance?

Answer

[Ans.: 0.2 Ω - 100.5A - 0.707]

Torque of Induction Motor under Running Condition (T_r)

- It is the torque developed by the motor under running condition (i.e., at slip *s*). From figure, the rotor circuit/phase at slip *s*.



- Let,

  $E_2^\backslash = sE_2$: rotor e.m.f/phase under running condition (*volts*)
  $X_2^\backslash = sX_2$: rotor reactance/phase under running condition (*Ohm*)
  R_2 : rotor resistance/phase under running condition (*Ohm*)

$Z_2^\backslash = \sqrt{R_2^2 + (sX_2^\backslash)^2}$: rotor impedance/phase running condition (*Ohm*)

Rotor current/phase, $I_2^\backslash = \dfrac{E_2^\backslash}{Z_2^\backslash} = \dfrac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$ … under running conditions

Rotor *p.f*, $\cos\phi_2^\backslash = \dfrac{R_2}{Z_2^\backslash} = \dfrac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$ … under running conditions

Torque of Induction Motor under Running Condition (T_r) …

- Then the motor torque/phase at slip *s* (or under running conditions) is:

$$T_r \ \alpha \ \Phi \ I_2^{\backslash} \cos\phi_2^{\backslash}$$

$$\alpha \ \Phi \ \frac{E_2^{\backslash}}{Z_2^{\backslash}} \frac{R_2}{Z_2^{\backslash}}$$

$$\alpha \ \Phi \ \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} \qquad \because E_2 \ \alpha \ \Phi$$

$$\alpha \ E_2 \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$T_r = \frac{K\,sE_2^2 R_2}{R_2^2 + (sX_2)^2} = \frac{K\,sE_2^2 R_2}{Z_2^{\backslash 2}} \qquad \underline{N_s \ is \ synch.\ speed \ (rps)}$$

$$if \ \ K = \frac{1}{2\pi N_s} \rightarrow T_r = \frac{1}{2\pi N_s} \cdot \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \ \ ....torque \ per \ phase$$

$$if \ \ K = \frac{3}{2\pi N_s} \rightarrow T_r = \frac{3}{2\pi N_s} \cdot \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \ \ ....motor \ torque \ under \ running \ condition$$

Torque of Induction Motor under Running Condition (T_r) …

- If the stator supply voltage V is constant, then stator flux and hence E_2 will be constant.

$$T_r = \frac{K s E_2^2 R_2}{R_2^2 + (sX_2)^2} = \frac{K_1 s R_2}{R_2^2 + (sX_2)^2}$$

Where: the constant K not equal to K_1

- It may be seen that running torque is:

i- It is directly proportional to the slip. If the slip increases (*i.e.*, motor speed decreases), the torque will increase and vice-versa.

ii- directly proportional to square of supply voltage: $\because E_2 \ \alpha \ V$

Maximum Torque under Running Conditions ($T_{r(max)}$)

- To get the maximum value of the running torque, take a derivative for it w.r.t the constant R_2 and equating the result with zero as follows:

$$\because T_r = \frac{K_1 s R_2}{R_2^2 + (sX_2)^2}$$

$$\therefore \frac{dT_r}{dR_2} = K_1\left[\frac{(s)\times(R_2^2+(sX_2)^2)-(2R_2)\times(sR_2)}{(R_2^2+(sX_2)^2)^2}\right] = 0$$

$$\therefore sR_2^2 + s(sX_2)^2 - 2sR_2^2 = 0$$

$$\therefore R_2^2 = (sX_2)^2$$

$$\therefore R_2 = sX_2 \qquad \textit{i.e., } s = \frac{R_2}{X_2} \textit{ under running condition}$$

Hence the running torque will be maximum when:

*Rotor resistance/phase = slip * Standstill rotor reactance/phase*

	$T_{r(\max)} = \frac{K_1 s}{2R_2}$	or	$T_{r(\max)} = \frac{K_1}{2X_2}$	

Maximum Torque under Running Conditions ($T_{r(max)}$)

- Also:

$$T_{r(\max)} = \frac{K_1}{2X_2} = \frac{KE_2^2}{2X_2} = \frac{3}{2\pi N_s}\frac{E_2^2}{2X_2} \qquad \textit{Max. motor torque under running condition}$$

Notes:

- *The maximum running torque is independent of rotor resistance.*
- *The maximum running torque varies inversely with the standstill reactance. Hence, it should be kept as small as possible.*
- *The maximum running torque varies directly with the square of the applied voltage.*